Math 4 Honors
Lesson 8-1: *What is the Total?*

Name ______________________________
Date ______________________________

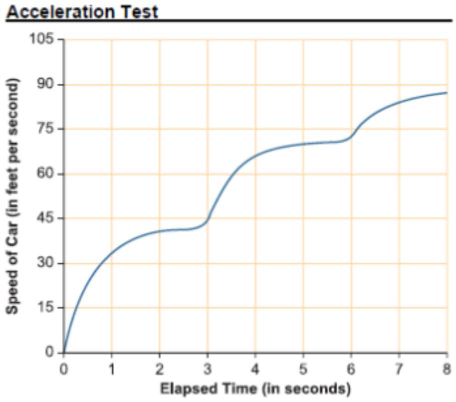
Learning Goal:

- *I can use a rate of change graph to calculate total change over some time interval.*

Some motion detectors are designed to produce tables and graphs of (*time*, *speed*) data rather than (*time*, *distance*) data. For example, police radar scanners focus on the speed of approaching cars, and baseball "radar guns" report the speed of pitched balls.

When automobile companies promote their new cars, one feature mentioned often is acceleration or "pickup." A standard statistic for measuring pickup is the time it takes to go from 0 to 60 miles per hour (88 feet per second).

The following graph shows the performance of one car during such an acceleration test.



Think About This Situation

The graph shows how the speed of an accelerating car changed over the 8-second time period it took to reach 60 miles per hour or 88 feet per second.

Discuss; not necessary to write answers

a. What does the pattern in the graph tell about results of the acceleration test?

b. At what point in the test was the car's speed greatest? At what point was the car accelerating most rapidly?

c. What is a good estimate of the distance traveled during the test and how would you convince someone else that your estimate is a good one?

If a function tells the position of a moving object at any time, the derivative of that function tells the instantaneous rate of change in position or velocity at any time. But the graph of speed for an accelerating car poses the inverse problem—given a function that tells velocity at any time, how can we find the position?

As you work on the problems of this investigation, look for answers to this question:

How can the information provided by a rate of change graph be used to calculate total change over some time interval?

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Think About This Situation

a. The speed-time graph is always increasing, but the rate of increase (acceleration) is not constant. The speed of the car continues increasing until it reaches about 88 ft/sec. The rate of increase in the speed follows a pattern of rapid increase then slower increase three times.

b. The car appears to be going fastest at the end of the test, about 88 feet per second; it appears to be accelerating (gaining speed) most rapidly right at the beginning of its trip. (There are other smaller accelerating bursts just before 3 seconds and just before 6 seconds. You might ask students why those later acceleration bursts occur. One plausible explanation is that it is at those points where the car shifts to a higher gear.)

c. Students might not have any clear ideas about how to estimate the total distance traveled in the 8-second test. The best they might come up with would be to estimate an average speed and multiply it by 8 seconds. They might suggest summing interval estimates such as 1 second in a way analogous to the derivative estimates in Lesson 1. If they do so, ask them how they arrive at their estimate of average speed. In fact, the definite integral can be used to calculate the average value for a function over some interval as shown below.

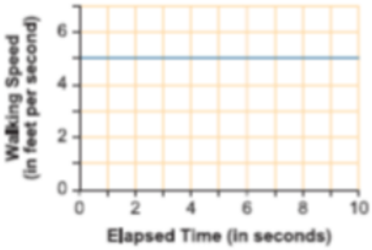
$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

1. Unlike cars, motorcycles, or airplanes, human walkers and runners can reach top speed quickly. They can also change speed and direction easily. The result will be graphs of (*time*, *speed*) data that are simpler than that of the 0–60 acceleration test.

Study the following (*time*, *speed*) graphs for students walking between classes in school. For each graph:

- describe (in terms of speed and acceleration) how you would walk to produce a similar graph.
- estimate the total distance the walker would have covered in the 10 seconds for which speeds are shown on the graph.

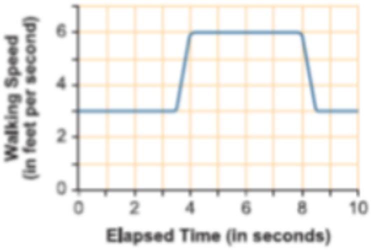
a. Patricia's Walk



Description:

Estimate:

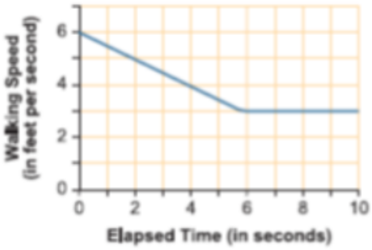
b. Raymond's Walk



Description:

Estimate:

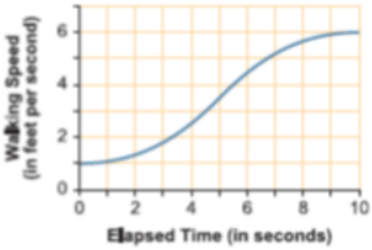
c. Fraser's Walk



Description:

Estimate:

d. Ayana's Walk



Description:

Estimate:

1 a. Patricia walks at a constant speed of 5 feet per second (no acceleration). The total distance traveled in 10 seconds is 50 feet.

b. Raymond varies his walking rate—about 3 feet per second for the first 3.5 seconds, accelerating quite quickly (in about 0.5 seconds) to 6 feet per second for the next 4 seconds, and then slowing rather quickly (in about 0.5 seconds) to 3 feet per second for the final 1.5 seconds. Students might deal with the half-second acceleration and deceleration periods in different ways (they might estimate an average speed midway between max and min on those intervals). The total distance estimates should be close to 43.5 feet.

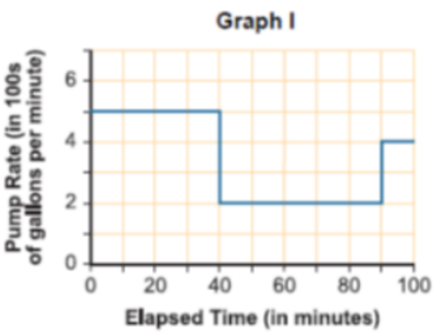
c. Fraser starts off walking at a brisk pace of 6 feet per second, but slows (decelerates) steadily to 3 feet per second after 6 seconds. Then he walks at a constant speed of 3 feet per second for the final 4 seconds of the walk. This leads to a distance estimate of $4.5(6) + 4(3) = 39$ feet.

d. Anaya starts off walking at 1 foot per second, then accelerates at an increasing rate until about 5 seconds into the walk. At that time, she continues to walk faster (reaching 6 ft/sec), but slows her rate of increase of speed. The total distance walked is about 35 feet.

2 People, cars, planes, and baseballs are not the only things in the world for which rate of change is an important measured statistic. Electricity, water, natural gas, oil, and computer data flow through a variety of "pipelines." The flow rates are monitored by gauges that report in units like gallons, cubic meters, or megabytes per second.

Study the following graphs that show recorded flow rates for pumps used to irrigate fields of a large farm. In each case, estimate the total amount of water pumped onto the fields in the indicated time intervals. Compare your results with those of others and resolve any differences.

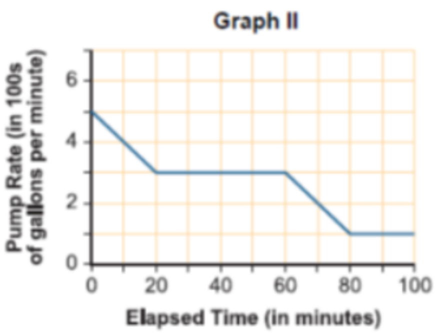
a.



Estimate & explanation:

34,000 gallons

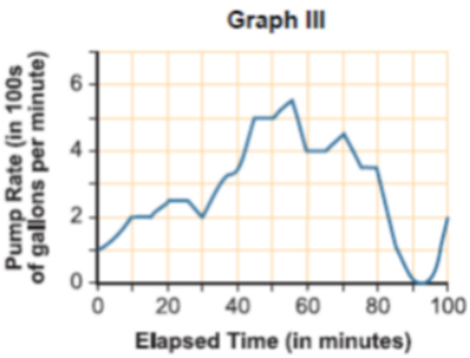
b.



Estimate & explanation:

26,000 gallons

c.



Estimate & explanation:

≈28,000 gallons

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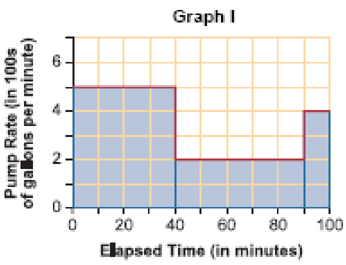
Teaching Resources

2 **INSTRUCTIONAL NOTE** In this problem, the context has changed but students are investigating the same ideas. You can expect your students to begin to make the connection between area under the rate curve and change in a quantity. Students who do make this connection may estimate area using more exact geometric figures such as triangles, rectangles, or trapezoids, or they may count the squares in the region. You may wish to have students share their methods for Graph iii.

a. 34,000 gallons

During the first 40 minutes, water is pumped at a rate of 5 hundred gallons per minute. Therefore, $5 \times 40 = 200$ hundred gallons (20,000 gallons) were pumped during that period. During the next 50 minutes, water is pumped at 2 hundred gallons per minute. During those 50 minutes, $2 \times 50 =$ 100 hundred gallons were pumped. Proceeding similarly, during the remaining 10 minutes, $4 \times 10 = 40$ hundred gallons were pumped.

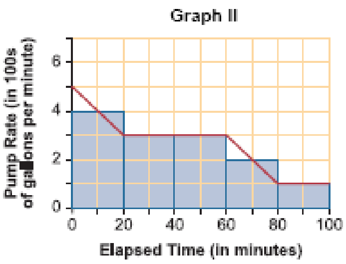
A reasonable estimate is $(5 \times 40) + (2 \times 50) + (4 \times 10) = 340$ hundred, or 34,000 gallons. This is the same as calculating the areas of the rectangles on the graph above.



b. 26,000 gallons

As before, there is more than one way to estimate the total amount of water pumped into the fields. One method using 20-minute intervals and the value of the flow rate at the midpoint is shown below.

Total amount pumped $= (4 \times 20) + (3 \times 20) + (3 \times 20) + (2 \times 20) + (1 \times 20) = 260$ hundred gallons, or 26,000 gallons.



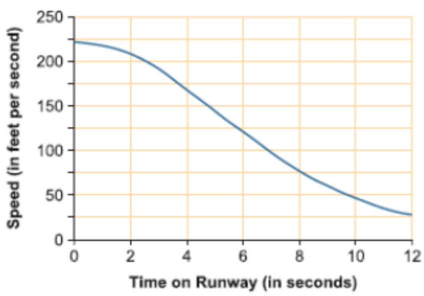
c. Estimating the total flow that results from this quite uneven flow rate graph is trickier, but it also illustrates an important need for what will become the definite integral. Students might choose to make an eye-ball estimate of a single average flow rate and from that make an estimate of total flow. However, you should push them by asking how they know that they have estimated average flow rate accurately. Suggest that they try to approximate the jagged graph by something like the graphs in Part b or in Parts b and d of Problem 1. They should come up with a total flow of about 28,000 gallons. The important point is not to get an exact answer, but to have a strategy mapped out that could produce quite an accurate estimate if required.

NOTE The solution to the Summarize the Mathematics is on page T529.

Check Your Understanding

When an airplane touches the runway after a flight, it has to slow down fairly quickly from its landing speed that might be about 150 mph or 220 feet per second to a taxiing speed of 20 mph or about 30 feet per second.

The speed graph for such a plane might look like this.



Use the graph to estimate the distance traveled by the plane during:

a. the first 6 seconds after landing.

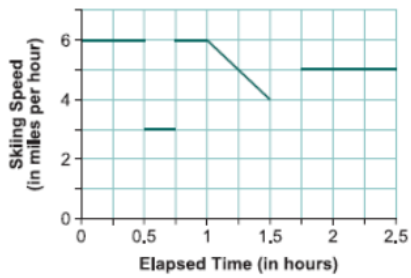
b. the time from 6 to 12 seconds after landing.

a. Answers in the vicinity of 1,100 feet are reasonable.

b. Answers in the vicinity of 400 feet are reasonable.

1 The following graph shows the reported speed of a cross-country skier during a 2.5-hour trip.

a. Describe the up and down contours of a ski trail that might lead to such a speed graph.



b. How far did the skier travel in the first hour? In the second hour?

c. How far did the skier travel in the time shown on the graph?

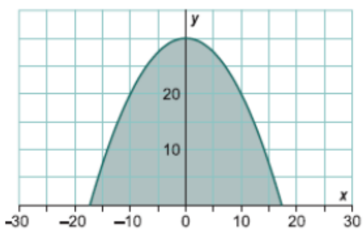
1 **a.** The given graph suggests a course that might be like this: A relatively level stretch that is covered during the first half-hour. A fifteen-minute stretch that might be somewhat uphill. Another fifteen-minute stretch on relatively level ground. A half-hour stretch that is gradually getting steeper. A 15-minute stop for a rest. And finally, a 45-minute stretch on relatively level ground where the skier is moving a bit more slowly than earlier on level ground—perhaps tiring at the end of his/her trek.

b. $3 + 0.75 + 1.5 = 5.25$ miles in the first hour; 3.75 miles in the second hour

c. Total of $5.25 + 3.75 + 2.5 = 11.5$ miles

3 A well-known church in Rockville, Maryland, has four large windowed arches that can be modeled by the function $h(x) = 30 - 0.1x^2$ with measurements in feet.

Use the window outline and grid shown to estimate the total area of glass used in each arch. Show the calculations used in making your estimate and explain how you could improve the accuracy of the estimate.



3 The actual window area given by the definite integral of the quadratic function from $-\sqrt{300}$ to $\sqrt{300}$ is approximately 693 square feet. However, if students use the fact that each full grid square has area 50 square feet and then try to piece together partial squares, they will not come to that exact value. Answers within 50 square feet of 700 square feet seem quite plausible. However, you might want to compare student answers to the exact value and use that as an opportunity to have a discussion about how to sharpen the estimates by refining the grid behind the window's graph.